Name:_____________________________ Date:___________________________

SOLUTION for Pre-Calculus 11 HW 4.5 Discriminant Nature of the Roots $D = b^2 - 4ac$

1. Determine the nature of the roots [ie: Determine how many x-intercepts each quadratic equation has]

a) $x^2 + 5x + 6 = 0$ To find the nature of the roots, just use the discriminant formula: $D = b^2 - 4ac$ $D = 25 - 4(1)(6)$ $D = 1$ There are two distinct real roots	b) $12x^2 + 7x - 3 = 0$ $D = b^2 - 4ac$ $D = 49 - 4(12)(-3)$ $D = 49 + 144 = 193 > 0$ There are two distinct real roots	c) $-2x^2 - 7x + 5 = 0$ $D = b^2 - 4ac$ $D = 49 - 4(-2)(5)$ $D = 89 > 0$ There are two distinct roots
d) $4x^2 = 13x - 8$ $4x^2 - 13x + 8 = 0$ $D = b^2 - 4ac$ $D = 169 - 4(4)(8)$ $D = 41 > 0$	e) $x(7 - 8x) = 10$ $7x - 8x^2 = 10$ $0 = 8x^2 - 7x + 10$ $D = b^2 - 4ac$ $D = 49 - 4(8)(10)$ $D = -271 < 0$ No real roots	f) $x(x+2) = 6 - (x-3)(2x+1)$ $x^2 + 2x = 6 - (2x^2 - 5x - 3)$ $x^2 + 2x = 6 - 2x^2 + 5x + 3$ $3x^2 - 3x - 9 = 0$ $D = b^2 - 4ac$ $D = 9 - 4(3)(-9)$ $D = 117 > 0$ There are two distinct roots

2. Solve each of the following inequalities:

a) $x^2 < 16$ $-4 < x < 4$	b) $x^2 - 25 > 0$ $x^2 > 25$ $x < -5 \text{ or } 5 < x$	c) $x(3 - x) < 0$ $x(3 - x) = 0$ $x = 0, x = 3$ $x(3 - x) < 0$ $x < 0 \text{ or } 3 < x$

3. Determine the value of "k" so that the equation has two equal roots:

a) $x^2 + kx + 25 = 0$ To have two equal roots, the discriminant must be equal to zero. $D = b^2 - 4ac$ $k^2 - 4(1)(25) = 0$ $k^2 = 100$ $k = \pm 10$	b) $kx^2 + 4x + 1 = 0$ $D = b^2 - 4ac$ $16 - 4(k)(1) = 0$ $16 = 4k$ $4 = k$	c) $0.5x^2 + 3kx + (3k - 4) = 0$ $D = b^2 - 4ac$ $(3k)^2 - 4(0.5)(3k - 4) = 0$ $9k^2 - 2(3k - 4) = 0$ $9k^2 - 6k + 8 = 0$ $(3k - 4)(3k + 2) = 0$ $k = \frac{4}{3} \text{ or } k = \frac{-2}{3}$

4. Determine the value of "k" so that the equation has two different roots:

a) $x^2 - kx + 12 = 0$	b) $kx^2 - kx + 1 = 0$	c) $x^2 - 4kx + (5k-6) = 0$
To have two distinct roots, the discriminant must be greater than 0. $b^2 - 4ac > 0$ $k^2 - 4(12) > 0$ $k^2 - 48 > 0$ $k < -4\sqrt{3} \quad or \quad 4\sqrt{3} < k$ Draw a number line and use test points:	$b^2 - 4ac > 0$ $k^2 - 4(k) > 0$ $k(k-4) > 0$ $k < 0 \quad or \quad 4 < k$	$b^2 - 4ac > 0$ $(-4k)^2 - 4(1)(5k-6) > 0$ $16k^2 - 20k + 24 > 0$ $4(k^2 - 5k + 6) > 0$ $4(k-2)(k-3) > 0$ $k < -2 \quad or \quad 3 < k$

5. Determine the value of "k" so that the equation has no real roots:

a) $x^2 - kx - 24 = 0$	b) $kx^2 - kx + 8 = 0$	c) $x^2 - 3kx - (3k-8) = 0$
To have no real roots, the discriminant must be less than 0 $b^2 - 4ac < 0$ $k^2 - 4(-24) < 0$ $k^2 + 96 < 0$ The left side is always positive, because k^2 is always positive. SO, no matter what "k", the equation will always have 2 distinct roots	$b^2 - 4ac < 0$ $k^2 - 4(k)(8) < 0$ $k^2 - 32k < 0$ $k(k-32) < 0$ $0 < k < 32$ if "K' is between 0 and 32, the equation will not have any roots	$x^2 - 3kx - 3k + 8 = 0$ $b^2 - 4ac < 0$ $(-3k)^2 - 4(-3k+8) < 0$ $9k^2 + 12k - 32 < 0$ $(3k+8)(3k-4) < 0$ $\frac{-8}{3} < k < \frac{4}{3}$

6. In order for a quadratic function to be factorable, what value must the discriminant be equal to? Explain:

This is the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. In order for a quadratic equation to be factorable, both roots must be either an integer or a fraction. Can't have a radical. So that means the discriminant $b^2 - 4ac$ needs to be either 0 or a perfect square.

If the quadratic equation $(x-2)^2 + k = 0$ has two distinct real roots, then what is the range of "k"? (Multiple choice, circle one) Justify your answer.

a) $k > 2$ b) $k < 0$ c) $k \leq 0$ d) $k \leq 4$

$$x^2 - 4x + 4 + k = 0$$
$$16 - 4(1)(4+k) > 0$$
$$16 - 4(4+k)$$
$$16 - 16 - 4k > 0$$
$$-4k > 0$$
$$k < 0$$

So as long as k<0, the quadratic equation will have two distinct roots

